1) Two satellites are in circular orbits around the Earth. The mass of satellite 1 is two times the mass of satellite 2. The orbital radius of satellite 2 is four times the orbital radius of satellite 1. Which of the following correctly compares the orbital speeds, v_1 and v_2, of the satellites?

(A) $4v_2 = v_1$ (B) $2v_2 = v_1$ (C) $v_2\sqrt{2} = v_1$ (D) $v_2 = v_1$

The free body diagrams for both satellites have only the force of gravity acting toward the center of mass of the Earth which is the in-direction. We can now sum the forces in the in-direction on both satellites and solve for each tangential speed.

$$F_g = \frac{Gmm}{r^2} \;\&\; \sum F_{in_{(\text{on }1)}} = F_{g1} = \frac{Gm_1m_E}{{R_1}^2} = m_1a_c = m_1\left(\frac{{v_1}^2}{R_1}\right)$$

Everybody brought m_1 and the inverse of R_1 to the party!

$$\Rightarrow \frac{Gm_E}{R_1} = {v_1}^2 \Rightarrow v_1 = \sqrt{\frac{Gm_E}{R_1}}$$

$$\& \sum F_{in_{(\text{on }2)}} = F_{g2} = \frac{Gm_2m_E}{{R_2}^2} = m_2a_c = m_2\left(\frac{{v_2}^2}{R_2}\right) \Rightarrow \frac{Gm_E}{R_2} = {v_2}^2 \Rightarrow v_2 = \sqrt{\frac{Gm_E}{R_2}}$$

And, because we know the relationship between the orbital radii, we can solve for the relationship between the orbital speeds.

$$R_2 = 4R_1 \Rightarrow v_2 = \sqrt{\frac{Gm_E}{4R_1}} = \frac{1}{2}\sqrt{\frac{Gm_E}{R_1}} = \frac{1}{2}v_1 \Rightarrow 2v_2 = v_1$$

The correct answer is B.

Please remember the mass of a satellite does not affect the orbital speed of the satellite, because everybody brings the mass of the satellite to the party!

2) On the surface of a planet with radius R and mass M, the gravitational field strength is g. Which of the following is the gravitational field strength at an altitude of 2R above the planet?

(A) 0 (B) $\frac{g}{2}$ (C) $\frac{g}{4}$ (D) $\frac{g}{9}$

$$F_g = m_{\text{object}}g = \frac{Gm_{\text{object}}m_{\text{planet}}}{r^2} \Rightarrow g = \frac{Gm_{\text{planet}}}{r^2} \Rightarrow g_{\text{at surface}} = g = \frac{GM}{R^2}$$

Everybody brought m_{object} to the party!

$$\Rightarrow g_{\text{at 2R}} = \frac{GM}{(R+2R)^2} = \left(\frac{1}{9}\right)\frac{GM}{R^2} = \frac{g}{9}$$

The correct answer is D.

Please realize "altitude" means height above sea level, or in this case, the height above the surface of the planet. Therefore, the distance between the centers of mass of the planet and the location 2R above the surface of the planet is R + 2R. A common mistake is to take r to be the distance from the surface of the planet or 2R. That would result in the following incorrect solution.

$$\Rightarrow g_{\text{incorrect}} = \frac{GM}{(2R)^2} = \left(\frac{1}{4}\right)\frac{GM}{R^2} = \frac{g}{4}$$

Again, g/4 is not the gravitational field 2R above the surface of the planet.
g/4 is the gravitational field a distance R above the surface of the planet.